

fields in producing power absorption in various subjects differs according to their sizes.

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## Characterization of Nonlinearities in Microwave Devices and Systems

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*Invited Paper*

**Abstract**—A simple model to describe a nonlinear device or system is proposed which extends the power series expansion, conventionally restricted to amplitude nonlinearities, to include phase nonlinearities as well. Four different test methods are selected for which the experimentally observed nonlinearity parameters are related to the "gain" and "phase" coefficients of the extended series. A set of simplified relationships is derived where the "1-dB gain compression point" represents gain contributions only while phase nonlinearities are included in the "intercept point," the "third-order intermodulation (IM) coefficients," and the "noise-power-ratio (npr)." For a TWT amplifier in which phase nonlinearities dominate, the third-order IM coefficient was measured. The results are compared with those calculated from single-tone and noise-loading tests using the relationships derived from the model. Agreement to  $\pm 1$  dB is found over a 15-dB power range.

#### I. INTRODUCTION

WITH microwave devices and systems utilized ever closer to their limits, linear measurement techniques are no longer sufficient to describe final performance under multisignal loading conditions. As a result, a number of techniques have evolved which are used to characterize nonlinear behavior and the resulting intermodulation (IM) performance. Selection of a particular technique depends on the type of information desired, such as detailed diagnostic information on the origin of nonlinearity, overall IM performance under different loading conditions, etc. Four of these

techniques—single-tone, two-tone, three-tone testing, and noise loading—were discussed at a panel session [1] on which this paper is based.

Some of the present microwave techniques have been adapted from the CATV industry [1, p. 112], [3] where the IM performance at UHF frequencies has been a primary concern for about two decades. There it has been found that the reliability of IM testing for system evaluation increases as the probing signal spectrum approaches that of the actual system load.

The usefulness of tests with probing signals which have a spectral distribution different from that of the final system load depends in part on how closely the selected mathematical model approaches actual device behavior. The Volterra series expansion [3]-[6] allows detailed and accurate representation of device characteristics, including memory, which can be applied directly to any spectral distribution of the system load. Measurement [6] of the relevant parameters (kernels), however, is sometimes time consuming and may exceed available measurement capabilities.

In this paper a simple mathematical model is proposed which is used to describe the amplitude and phase nonlinearities (gain deviation and AM-PM conversion) observed in microwave devices. From this model the parameters relevant to each of the four measurement techniques are derived and interrelated (equation numbers of simplified relationships are marked  $\square$  for convenient reference). For each technique

a representative experimental setup is shown schematically, and the limitations of the measured parameters are discussed. For a selected device [a traveling wave tube (TWT)], predicted and observed parameters are compared.

## II. THE MODEL

Selection of a model [2]–[10] was based on combining analytical simplicity with experimental convenience in determining parameter values to represent both amplitude and phase nonlinearities. Such a model can be constructed by extending the conventional amplitude power series expansion [2], [11] to include order-dependent time delays (Fig. 1). For the case when time delays of all orders are equal, this model reduces to the conventional amplitude model; for unequal time delays, power-level-dependent phase shifts are introduced. Consider, for example, the total output in the fundamental band. It consists of the output due to the linear term (“linear output”) and the fundamental components of all odd-order terms. Relative to the linear output, these have in-phase and quadrature-phase components where the angle of their vector sum depends on the time-delay differences.

The time-dependent voltage transfer function between the input and output of the device is, therefore, written as

$$e_o(t) = c_0 + c_1 e_i(t - t_1) + c_2 e_i^2(t - t_2) + c_3 e_i^3(t - t_3) + \dots \quad (1)$$

where

- $e_o(t)$  instantaneous output voltage [V];<sup>1</sup>
- $e_i(t)$  instantaneous input voltage [V];
- $c_j$  series expansion coefficient of order  $j$  [V $^{-j+1}$ ];
- $t_j$  time delay of the  $j$ th-order term [s].

Here  $c_j$  is assumed to be independent of frequency and power level. All time-delay differences are referenced to the delay of the linear output

$$\Delta t_j = t_j - t_1. \quad (2)$$

In many practical cases devices operate with less than octave bandwidth. Since even-order terms in (1) will produce distortion products which fall outside this band, subsequent discussions will neglect all even-order terms. Only inband IM components produced by odd-order terms will be considered.

## III. SINGLE-TONE TEST

Although this test is not now widely used, it provides a simple means of separating amplitude and phase nonlinearity contributions and allows estimating the IM performance under more complicated loading conditions. The accuracy of this estimate depends on how closely (1) models the device; good agreement has been found, for example, in TWT amplifiers.

In the single-tone test, the device is excited with

$$e_i(t) = A \cos(\alpha t + \phi_0) \quad (3)$$

where  $A$  equals peak input voltage [V] and  $\phi_0 = 0$  from selection of input reference plane. With only odd-order terms retained in (1), this results in an output voltage of

$$e_o(t) = c_1 A \cos(\alpha(t - t_1) + c_3 A^3 \cos^3 \alpha(t - t_3) + \dots \quad (4)$$

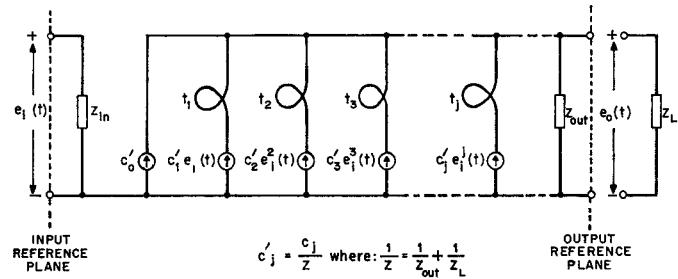


Fig. 1. Model of nonlinearity.

If all output measurements are referenced to the time-delayed ( $\tau_D = t_1$ ) input signal, we can set  $t_1 = 0$ . Using trigonometric identities and collecting equal frequency terms, the output voltage at frequency  $n\alpha$  can be written as

$$e_{on\alpha}(t) = A_{n\alpha} \cos(n\alpha t) + B_{n\alpha} \sin(n\alpha t), \quad n = \text{odd.} \quad (5)$$

It is convenient to write the amplitudes  $A_{n\alpha}$  and  $B_{n\alpha}$  of the in-phase and quadrature-phase components of (5) in terms of new gain and phase coefficients  $a_j$  and  $b_j$ . At frequency  $n\alpha$  these are related to the coefficients  $c_j$  of (1) by

$$\begin{aligned} a_j^{(n\alpha)} &= c_j \cos(n\alpha \Delta t_j) \\ b_j^{(n\alpha)} &= c_j \sin(n\alpha \Delta t_j) \end{aligned} \quad n, j = \text{odd.} \quad (6)$$

Using (2) and with  $t_1 = 0$ , the fundamental and third-harmonic amplitudes become

$$A_\alpha = a_1^{(\alpha)} A + \frac{3}{4} a_3^{(\alpha)} A^3 + \frac{5}{8} a_5^{(\alpha)} A^5 + \frac{35}{64} a_7^{(\alpha)} A^7 + \dots \quad (7a)$$

$$B_\alpha = b_1^{(\alpha)} A + \frac{3}{4} b_3^{(\alpha)} A^3 + \frac{5}{8} b_5^{(\alpha)} A^5 + \frac{35}{64} b_7^{(\alpha)} A^7 + \dots \quad (7b)$$

$$A_{3\alpha} = \frac{1}{4} a_3^{(3\alpha)} A^3 + \frac{5}{16} a_5^{(3\alpha)} A^5 + \frac{21}{64} a_7^{(3\alpha)} A^7 + \dots \quad (8a)$$

$$B_{3\alpha} = \frac{1}{4} b_3^{(3\alpha)} A^3 + \frac{5}{16} b_5^{(3\alpha)} A^5 + \frac{21}{64} b_7^{(3\alpha)} A^7 + \dots \quad (8b)$$

Note that from the above assumptions

$$a_1^{(\alpha)} = c_1 \quad b_1^{(\alpha)} = 0. \quad (9)$$

For the present test, only the fundamental output is considered ( $n = 1$ ); in subsequent discussions the superscript  $(\alpha)$  in (6), (7), and (9) is omitted for convenience. For the fundamental component, (5) can be written as

$$e_{o\alpha}(t) = (\hat{e}_{o\alpha}) \cos(\alpha t + \phi_\alpha) \quad (10)$$

where

$$\hat{e}_{o\alpha} = \sqrt{A_\alpha^2 + B_\alpha^2} \approx A_\alpha \quad (11a)$$

$$\phi_\alpha = \arctan \frac{B_\alpha}{A_\alpha} \approx \frac{B_\alpha}{A_\alpha}. \quad (11b)$$

The approximations in (11) are valid at sufficiently low power levels or high gains where  $A_\alpha \gg B_\alpha$  (since  $b_1 = 0$ ), which in many devices extends into the region where gain changes can readily be observed.

Single-tone tests are conveniently performed in a bridge-type circuit (Fig. 2). The CW source is low-frequency modu-

<sup>1</sup> Dimensions are denoted by square brackets throughout.



$$A_{2\alpha \pm \beta} = \frac{3}{4} a_3^{(2\alpha \pm \beta)} A^3 + \frac{25}{8} a_5^{(2\alpha \pm \beta)} A^5 + \dots \quad (21c)$$

$$A_{3\alpha} = \frac{1}{4} a_3^{(3\alpha)} A^3 + \frac{25}{16} a_5^{(3\alpha)} A^5 + \dots \quad (21d)$$

and corresponding expressions for  $B_{m,n}$ . As in (6), the gain and phase coefficients are

$$\begin{aligned} a_j^{(n\alpha+m\beta)} &= c_j \cos(n\alpha + m\beta) \Delta t_j \\ b_j^{(n\alpha+m\beta)} &= c_j \sin(n\alpha + m\beta) \Delta t_j \end{aligned} \quad (m+n), \quad j = \text{odd} \quad (22)$$

with output amplitude, phase, and power levels as in (10)–(12).

The intercept point is found experimentally from either individual tone or total power measurements. Here, the former method is selected where the input and output power levels of one of the exciting tones (for example,  $p_{in,\alpha}$  and  $p_\alpha$ ) and one of the IM products (for example,  $p_{2\alpha-\beta}$ ) are measured [Fig. 3(a)]. The input power level  $p_{in,\alpha}$  from the two CW sources ( $\alpha$  and  $\beta$ ) set to equal levels is controlled by the input attenuator and monitored by the input power meter. The fundamental output power level is measured using a bandpass filter (BPF) with bandwidth  $B < |\alpha - \beta|/2$  around the center frequency  $\alpha$ . A second BPF with the same bandwidth around the center frequency  $2\alpha - \beta$  and a power meter measure the selected IM product. A high-quality isolator has to precede this filter to absorb the reflected fundamental power. The results are plotted [Fig. 3(b), solid lines] on logarithmic scales (for example, in dBm). The intercept point is then defined as that output power level  $P_I$  [dBm] at which  $P_{2\alpha-\beta}$  [dBm] would intercept  $P_\alpha$  [dBm] if low-level results were extrapolated [Fig. 3(b), dashed lines] into the high-power region.

To relate  $P_I$  to the coefficients of (22), assume that the test frequencies are selected such that  $\alpha \approx \beta \approx (2\alpha - \beta)$ ; therefore

$$a_j^{(\alpha)} \approx a_j^{(\beta)} \approx a_j^{(2\alpha-\beta)} \equiv a_j \quad (23a)$$

$$b_j^{(\alpha)} \approx b_j^{(\beta)} \approx b_j^{(2\alpha-\beta)} \equiv b_j. \quad (23b)$$

Then, by the definition of the extrapolations, the small angle approximations of (11) hold and  $A_\alpha = a_1 A$  such that from (12), (21), and (22)

$$p_\alpha = \left( \frac{a_1 A}{\sqrt{2}} \right)^2 \frac{10^3}{R} \quad [\text{mW}] \quad (24a)$$

$$p_{2\alpha-\beta} = \left( \frac{1}{\sqrt{2}} \frac{3}{4} A^3 \sqrt{a_3^2 + b_3^2} \right)^2 \frac{10^3}{R} \quad [\text{mW}]. \quad (24b)$$

Since at  $p_I$  by definition  $p_\alpha = p_{2\alpha-\beta}$

$$p_I = \frac{2}{3} a_1^2 \left( \frac{a_1}{\sqrt{a_3^2 + b_3^2}} \right) \frac{10^3}{R} \quad [\text{mW}] \quad (25a)$$

or, for  $R = 50 \Omega$  and using (13)

$$\begin{aligned} P_I &= 10 \log_{10} p_I \\ &= G_0 + 10 \log_{10} \left( \frac{a_1}{\sqrt{a_3^2 + b_3^2}} \right) \\ &\quad + 11.25 \quad [\text{dBm}]. \end{aligned} \quad (25b)$$

Note that if  $P_I$  were defined in terms of the total power levels

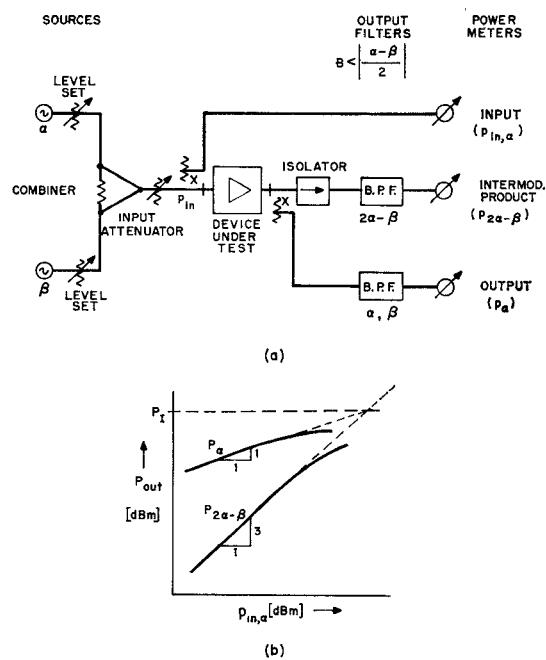


Fig. 3. Two-tone nonlinearity test. (a) Schematic measurement circuit. (b) Definition of the intercept point.

(i.e.,  $p_\alpha + p_\beta$ ), then the intercept point would shift to  $P_I' = P_I + 4.52 \text{ dBm}$ .

A relationship between  $P_I$  and  $P_{\alpha,1} \text{ dB}$  can be found from (14) and (25); for third-order devices in which  $a_3 \gg b_3$

$$P_I = P_{\alpha,1} \text{ dB} + 10.6 \quad [\text{dBm}]. \quad (26)$$

Note, however, that in many practical devices these assumptions are not valid. Note, also, that because of the difference in constants between (7) and (21), the 1-dB gain compression point found from a two-tone test and measuring total power output,  $P_{\text{total},1} \text{ dB}$ , is related to (14) by

$$P_{\text{total},1} \text{ dB} = P_{\alpha,1} \text{ dB} - 1.76 \quad [\text{dBm}]. \quad (27)$$

From (25) it is seen that  $p_I$  is independent of  $p_{in,\alpha}$ ; the intercept point is, therefore, a useful measure of the total device nonlinearity. From its knowledge and in the power region where a given device follows the extrapolation, the IM level is related to  $P_\alpha$  by

$$P_{2\alpha-\beta} = 3P_\alpha - 2P_I \quad [\text{dBm}]. \quad (28)$$

Caution has to be exercised, however, when these conditions are not met, for example, when amplitude or frequency-dependent linearization techniques are used [15], [16]. When cascading devices, the final device requires the highest  $P_I$ . Assume that there is no interaction between nonlinearities of neighboring devices (such as cancellation of phase contributions). Then from (25) the intercept point of the cascade is approximately equal to the intercept point of the final device if the intercept points of successive devices differ by less than the gain of each stage.

## V. THREE-TONE TEST

Here, again, specific inband IM products are selected to characterize overall device nonlinearities, commonly through the third-order intermodulation coefficient [9], [11], [17].

The more even spectral distribution and flexibility while still allowing discrete frequency evaluation make this an attractive test for multifrequency (such as communication) systems.

In this test three (equal-level) tones are applied to the input

$$e_i(t) = A(\cos \alpha t + \cos \beta t + \cos \gamma t) \quad (29)$$

yielding from (1) an output spectrum of the form

$$e_o(t) = \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e_{n\alpha+m\beta+l\gamma} \quad (30)$$

where

$$e_{n\alpha+m\beta+l\gamma} = A_{l,m,n} \cos(n\alpha + m\beta + l\gamma)t + B_{l,m,n} \sin(n\alpha + m\beta + l\gamma)t \quad (31)$$

with amplitudes

$$A_{\alpha} = a_1^{(\alpha)} A + \frac{15}{4} a_3^{(\alpha)} A^3 + \frac{155}{8} a_5^{(\alpha)} A^5 + \dots \quad (32a)$$

$$A_{\alpha\pm\beta\pm\gamma} = \frac{3}{2} a_3^{(\alpha\pm\beta\pm\gamma)} A^3 + \frac{45}{4} a_5^{(\alpha\pm\beta\pm\gamma)} A^5 + \dots \quad (32b)$$

$$A_{2\alpha\pm\beta} = \frac{3}{4} a_3^{(2\alpha\pm\beta)} A^3 + \frac{55}{8} a_5^{(2\alpha\pm\beta)} A^5 + \dots \quad (32c)$$

$$A_{2\alpha\pm 2\beta\pm\gamma} = \frac{15}{8} a_5^{(2\alpha\pm 2\beta\pm\gamma)} A^5 + \dots \quad (32d)$$

$$A_{3\alpha} = \frac{1}{4} a_3^{(3\alpha)} A^3 + \frac{45}{16} a_5^{(3\alpha)} A^5 + \dots \quad (32e)$$

$$\vdots \quad \vdots$$

and corresponding expressions for  $B_{l,m,n}$ . For brevity, only one expression for each type of output amplitude is given in (32). Corresponding expressions can be found by permutation of  $\alpha$ ,  $\beta$ , and  $\gamma$ .

The coefficients of order  $j$  are given by

$$a_j^{(n\alpha+m\beta+l\gamma)} = c_j \cos(n\alpha + m\beta + l\gamma) \Delta t_j \quad (33)$$

$$b_j^{(n\alpha+m\beta+l\gamma)} = c_j \sin(n\alpha + m\beta + l\gamma) \Delta t_j$$

$$(l + m + n), \quad j = \text{odd.}$$

In terms of the output power levels, defined as in (12), the third-order intermodulation coefficient is defined [11] as

$$m_x \equiv \frac{p_x}{p_{\alpha} p_{\beta} p_{\gamma}} \bar{p}^2 \quad (34a)$$

where

$\bar{p} = 1 \text{ mW}$  = normalization factor;

$p_x$  = measured output power level of the IM product at frequency  $x$  (e.g.,  $3\alpha$ ,  $\alpha + \beta - \gamma$ , etc.).

Second- or higher order IM coefficients with similar properties can be defined by analogy [11]. Note that if a third-order device is operated in the low-power region ( $A_{\alpha} \approx a_1 A$ ), then  $m_x$  is independent of power level and thus a convenient measure of overall device nonlinearity. For the equal-level excitation

(34a) can be written in logarithmic form as

$$M_x = 10 \log_{10} m_x = P_x - 3P_{\alpha} \quad [\text{dB}] \quad (34b)$$

since  $10 \log_{10} \bar{p} = 0 \text{ dBm}$ . With the assumption that in this test the narrow-band approximation of (23) still holds, the IM coefficients associated with the individual inband product frequencies are related from (32) by

$$M_{\alpha+\beta-\gamma} = M_{2\alpha-\beta} + 6 = M_{3\alpha} + 15.56 \quad [\text{dB}]. \quad (35)$$

The highest  $(\alpha + \beta - \gamma)$ -type product is frequently selected [17] for measuring  $M$  in a circuit of the type shown in Fig. 3(a). However, because of the more complex output spectrum in the region of the fundamental band of (30) as compared to (19), simple filtering of tones may not be sufficient to yield accurate results. Other techniques [5, fig. 5] (such as linear output tone cancellation [18]) may have to be employed.

From (12), (23), (32), and (34),  $m_{\alpha+\beta-\gamma}$  is related to the gain and phase coefficients in a third-order device ( $a_j, b_j = 0$ , for  $j > 3$ ) by

$$m_{\alpha+\beta-\gamma} = \left( \frac{3\bar{p}}{a_1^2 p_{\text{in}}} \right)^2 (\mathfrak{A}_3^2 + \mathfrak{B}_3^2) \cdot \left[ \frac{1}{\left( 1 + \frac{5}{2} \mathfrak{A}_3 \right)^2 + \left( \frac{5}{2} \mathfrak{B}_3 \right)^2} \right]^3 \quad (36)$$

where  $p_{\text{in}} = 3p_{\text{in},\alpha}$  equals total input power, and

$$\mathfrak{A}_3 \equiv \frac{a_3}{a_1} \frac{p_{\text{in}} R}{10^3} \quad \mathfrak{B}_3 \equiv \frac{b_3}{a_1} \frac{p_{\text{in}} R}{10^3}. \quad (37)$$

At low input power levels where  $\mathfrak{A}_3, \mathfrak{B}_3 \ll 1$ , the last bracket of (36) becomes unity and

$$m_{\alpha+\beta-\gamma} = \left( \frac{a_3^2 + b_3^2}{a_1^2} \right) \left( \frac{3\bar{p} R}{10^3 a_1^2} \right)^2 \quad (38a)$$

or, with (13) and for  $R = 50 \Omega$

$$M_{\alpha+\beta-\gamma} = 10 \log_{10} \left( \frac{a_3^2 + b_3^2}{a_1^2} \right) - 2G_0 - 16.5 \quad [\text{dB}]. \quad (38b)$$

With these assumptions,  $M_{\alpha+\beta-\gamma}$  can therefore be calculated from the gain and phase coefficients measured in a single-tone test. Introducing  $\oplus$  for "power addition" such that

$$10 \log_{10} \left( \frac{a_3^2 + b_3^2}{a_1^2} \right) = \left[ 10 \log_{10} \left( \frac{a_3}{a_1} \right)^2 \right] \oplus \left[ 10 \log_{10} \left( \frac{b_3}{a_1} \right)^2 \right] \quad (39)$$

then from (14) and (17)

$$M_{\alpha+\beta-\gamma} = [-2P_{\alpha,1 \text{ dB}} - 15.3] \oplus [20 \log_{10} k_0 - 89.1] \quad [\text{dB}]. \quad (40a)$$

Also, from (25b) and (38b)

$$M_{\alpha+\beta-\gamma} = -2P_I + 6 \quad [\text{dB}]. \quad (40b)$$

Note that when devices are cascaded and if there is no interaction between nonlinearities, the IM coefficient translates from (38b) with twice the gain of each section, whereas both  $P_{a,1}$  dB and  $P_I$  translate directly with gain (or loss).

When the low-level approximation does not hold, it is seen from (36) and (37) that  $M$  becomes unsymmetrical in  $a_3$  and  $b_3$  and depends on the sign of  $a_3$ ; for example, for a given  $|a_3|$  the value of  $M$  differs between expansive and compressive devices.

## VI. NOISE LOADING

In this test [7], [11], [19], [20] the input signal is obtained from a "white" noise source which is band limited to the instantaneous frequency range of interest. The nonlinearity to be measured is then given in terms of the noise-power-ratio (npr), which relates the "signal" noise output to the IM noise generated by all nonlinearities. For signals whose spectral distribution can be approximated by that of white noise, this test method is used commercially [19] to evaluate the IM performance of complete systems.

Let  $S_i(f)$ , the input power spectral density of the zero-mean Gaussian noise source, be a constant  $N_i$  over the selected bandwidth  $2\Delta f$  around the center frequency  $f_0$  [Fig. 4(a)]. Kuo [21] has shown that if (1) is extended to the  $J$ th order, and with the narrow-band assumptions of (23), then the auto-correlation function of the output signal is given by

$$R_{\text{out}}(\tau) = \sum_{n=0}^J (A_n + B_n) R_{\text{in}}^n(\tau). \quad (41a)$$

Here

$$A_n = \frac{1}{n!} \left[ \sum_{m=0}^L \frac{(n+2m)! a_{n+2m}}{2^m m!} R_{\text{in}}^m(0) \right]^2 \quad (41b)$$

with a similar expression for  $B_n$ , and

$$L = \begin{cases} \frac{J-n}{2}, & \text{for } J-n \text{ even} \\ \frac{J-n-1}{2}, & \text{for } J-n \text{ odd} \end{cases}$$

$$R_{\text{in}}(\tau) \equiv \overline{e_i(t)e_i(t+\tau)} \quad (\text{statistical average}) \quad (41c)$$

$$\begin{aligned} R_{\text{in}}(0) &\equiv \int_{-\infty}^{\infty} S_i(f) df = 4\Delta f N_i \\ &= p_{\text{in}}^{(N)} \frac{R}{10^3} \equiv \dot{p} \end{aligned} \quad (41d)$$

where  $p_{\text{in}}^{(N)}$ , defined as in (12), is the average "signal" noise input power, and  $\dot{p}$  is defined for notational convenience. The Fourier transform of (41a) then yields the signal output power spectral density  $S_o(f)$ .

To find the IM noise output, a very narrow portion of the input spectrum,  $\delta f \ll \Delta f$  (Fig. 4a), is removed ("notch" [11], "dark band" [19]). Then, the major contribution to the output power within the notch is due to nonlinearities. With the notch placed at  $f=f_0$ , the npr is defined as

$$\text{npr} = \frac{S_o(f_0)}{S_{on}(f_0)} \quad (42)$$

where  $S_{on}(f_0)$  is  $S_o(f_0)$  with the notch in place. Kuo [21] has shown that (for  $J=5$  and  $b_1=0$ )

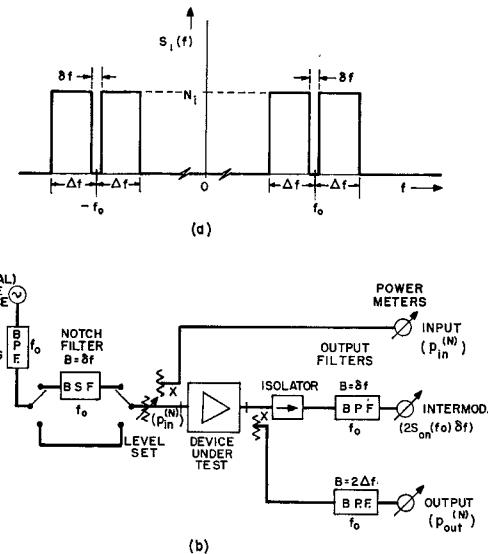


Fig. 4. Noise-loading test. (a) Signal noise input power density spectrum. (b) Schematic measurement circuit.

$$\begin{aligned} S_o(f_0) &= (a_1 + 3a_3\dot{p} + 15a_5\dot{p}^2)^2 N_i + (3b_3\dot{p} + 15b_5\dot{p}^2)^2 N_i \\ &+ 6[(a_3 + 10a_5\dot{p})^2 + (b_3 + 10b_5\dot{p})^2] (\frac{3}{4}\dot{p})^2 N_i \\ &+ \frac{2875}{64} (a_5^2 + b_5^2) \dot{p}^4 N_i \end{aligned} \quad (43a)$$

and, if the notch is sufficiently narrow, Kuo [21] has shown that for the case when the notch is in place

$$\begin{aligned} S_{on}(f_0) &\approx 6[(a_3 + 10a_5\dot{p})^2 + (b_3 + 10b_5\dot{p})^2] (\frac{3}{4}\dot{p})^2 N_i \\ &+ \frac{2875}{64} (a_5^2 + b_5^2) \dot{p}^4 N_i. \end{aligned} \quad (43b)$$

From (42) and (43)

$$\text{npr} \approx 1$$

$$+ \frac{(a_1 + 3a_3\dot{p} + 15a_5\dot{p}^2)^2 + (3b_3\dot{p} + 15b_5\dot{p}^2)^2}{\frac{27}{8} [(a_3 + 10a_5\dot{p})^2 + (b_3 + 10b_5\dot{p})^2] \dot{p}^2 + \frac{2875}{64} (a_5^2 + b_5^2) \dot{p}^4}.$$

$$(44)$$

As long as the narrow-band approximations are valid, (44) predicts the input power dependence of npr for a set of gain and phase coefficients which can be found from a single-tone test. A typical npr measurement circuit [11], [19], [20], [5, fig. 11] [Fig. 4(b)] uses an output power meter preceded by a BPF with a bandwidth equal to or slightly less than that of the bandstop filter (BSF), which introduces the notch. For a given input power level, the npr then is the ratio of the intermodulation power meter readings with and without the notch in place. Power level and gain (or loss) measurements are obtained from the input and output power meter readings. The purpose of the isolator is the same as in Fig. 3(a).

Consider now a third-order device ( $a_j, b_j=0$ , for  $j>3$ ). Then, for  $\dot{p}/a_1$  small, (44) can be rewritten using (37) as

$$\text{npr} = \frac{11}{3} + \frac{16}{9} \frac{1}{\mathcal{G}_3} + \frac{8}{27} \left( \frac{1}{\mathcal{G}_3^2 + \mathcal{G}_3^2} \right). \quad (45)$$

As expected,  $\text{npr} \rightarrow \infty$  as  $a_3, b_3 \rightarrow 0$ , that is, as the device approaches linearity. For small values of  $\mathcal{G}_3, \mathcal{G}_3$  (that is, low power levels or high gains), (45) can be approximated by

$$npr \approx \frac{8}{27} \left( \frac{10^3}{R} \right)^2 a_1^2 \left( \frac{1}{a_3^2 + b_3^2} \right) \left( \frac{1}{p_{in}^{(N)}} \right)^2, \\ \text{for } npr > 10^3 \quad (45a)$$

and thus  $(npr) (p_{in}^{(N)})^2$  approaches the "noise constant"  $n_r$ , which is characteristic of a given device; in logarithmic notation, with (13) and for  $R=50 \Omega$

$$N_r \equiv 10 \log_{10} (n_r) = \text{NPR} + 2P_{in}^{(N)} \quad [\text{dB}] \\ = G_0 + 10 \log_{10} \left( \frac{1}{a_3^2 + b_3^2} \right) + 20.74 \quad [\text{dB}] \quad (45b)$$

where, from (41d)

$$P_{in}^{(N)} = 10 \log_{10} p_{in}^{(N)} \\ = \text{average "signal" noise input power} \quad [\text{dBm}] \\ \text{NPR} = 10 \log_{10} (npr) \quad [\text{dB}].$$

This approximation can be used to find simple relationships between npr of (45a),  $m_{\alpha+\beta-\gamma}$  of (38), and  $\rho_I$  of (25) if it is assumed that the corresponding total average input power levels ( $p_{in}^{(N)}$ ,  $3p_{in,\alpha}$ , and  $2p_{in,\alpha}$ , respectively) are equivalent. Then

$$\rho_I = G_0 + \frac{1}{2}(\text{NPR} + 2P_{in}^{(N)}) + 0.87 \quad [\text{dBm}] \quad (46a)$$

and

$$M_{\alpha+\beta-\gamma} = -(\text{NPR} + 2P_{in}^{(N)}) \\ - 2G_0 + 4.26 \quad [\text{dB}]. \quad (46b)$$

As  $\alpha_3$  and  $\beta_3$  increase further, (45) has to be used. The selection of sign for the roots of the quadratic equation (45) depends again, as in (36) for  $m_{\alpha+\beta-\gamma}$ , on the character of the nonlinearity, that is, on whether the device is expansive or compressive. In this region gain deviations can no longer be neglected. As in (41d) the total inband "signal" noise power output  $p_o^{(N)}$  is given by

$$p_o^{(N)} \frac{R}{10^3} = R_o(0) \approx \sum_{f_o-\Delta f}^{f_o+\Delta f} S_o(f) df \approx 2S_o(f_o)(2\Delta f). \quad (47a)$$

From (37) and (43a) this becomes

$$p_o^{(N)} = a_1^2 p_{in}^{(N)} \left[ 1 + 6\alpha_3 + \frac{99}{8} (\alpha_3^2 + \beta_3^2) \right] \quad [\text{mW}] \quad (47b)$$

which, together with (45) and (36), defines the correspondence between  $m_{\alpha+\beta-\gamma}$  and npr. For convenience, the deviation  $\Delta M_{\alpha+\beta-\gamma}$  [dB] from the asymptotic expression (46b) is plotted as a function of NPR in Fig. 5.

## VII. COMPARISON WITH EXPERIMENTAL RESULTS

The validity of the model and the resulting conversion equations were evaluated for the case of a TWT amplifier. Three sets of measurements were performed over a relative bandwidth of  $2\Delta f/f_o = 0.5$  percent as a function of power level:

- 1) a single-tone test [12] at band center;
- 2) a three- (equal-level) tone test [22] with two frequencies ( $\alpha$  and  $\gamma$ ) located at the band edges, and the third frequency ( $\beta$ ) located  $\Delta f/5$  from the lower band edge;
- 3) a noise-loading test [20], [23] with a 10-dB notch width of  $\Delta f/200$  located at center band.

The gain and phase coefficients in (6) are extracted from

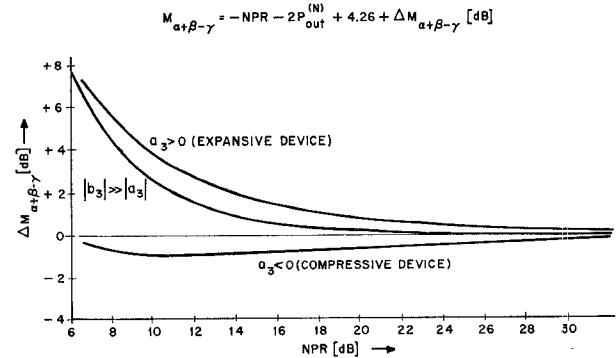


Fig. 5. Relation between  $M_{\alpha+\beta-\gamma}$  and NPR for third-order nonlinearities with correction curves for low values of NPR when either gain deviations ( $a_3 > 0$ ,  $a_3 < 0$ ) or phase deviations ( $|b_3| \gg |a_3|$ ) dominate.

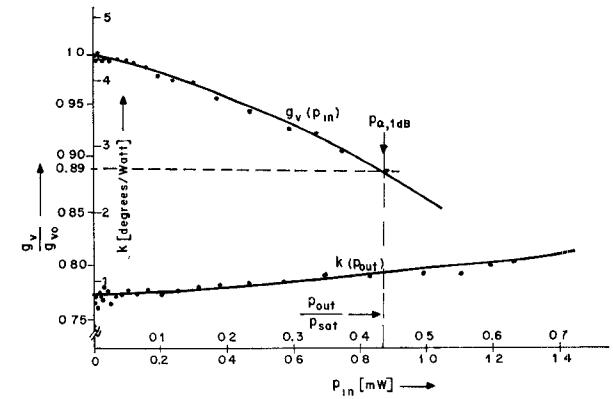


Fig. 6. Curve fitting (to fifth order) of single-tone measurements on a traveling-wave tube.

TABLE I

SINGLE-TONE GAIN AND PHASE COEFFICIENTS

j =	0	1	3	5	7
$a_j$	0	158.95	-283	$-2.45 \times 10^3$	$-1.28 \times 10^4$
$k_j$	0.81	$2.45 \times 10^{-2}$	$1.26 \times 10^{-3}$	0	0
$b_j$	0	0	757.0	$6.95 \times 10^3$	$5.64 \times 10^4$

curve fitting the single-tone test results to (7) and (15). The results are used in (36), (45), and (47b) to predict  $M_{\alpha+\beta-\gamma}$  and NPR. These predictions are then compared with the results of experiments 2) and 3).

In the single-tone test [12], the input power level was stepped in 1-dB increments over a power range of approximately 30 dB. At the output, the level of the fundamental tone  $P_\alpha$  [dBm], the change in power gain  $\delta G_\alpha$  [dB], and the change of insertion phase  $\delta \phi_\alpha$  [deg] were recorded for each step. For  $R=50 \Omega$ , the voltage gain  $g_v(p_{in})$  and the AM-PM conversion coefficient  $k(p_{out})$  were calculated. To the resulting points polynomials were fitted (Fig. 6) from which the single-tone coefficients were obtained (Table I).

Over the same dynamic range, a set of three-tone tests [17], [22] was performed (Fig. 7, open circles). In the output power range between +18 dBm and +30 dBm,  $M_{\alpha+\beta-\gamma}$  is found to be constant. At higher power levels  $|M_{\alpha+\beta-\gamma}|$  decreases due to compression of  $p_\alpha$ . This is consistent with the single-tone test observations: The major contribution to the observed behavior arises from AM-PM conversion as seen

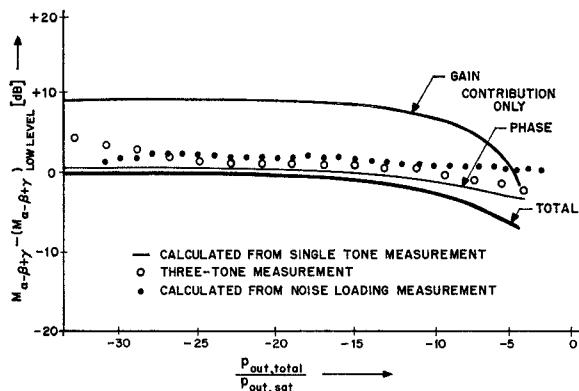


Fig. 7. Comparison of three-tone measurement results with  $M_{\alpha-\beta+\gamma}$  calculated from single-tone and noise-loading measurements using the conversion relations derived from the model.

from the curves (solid lines in Fig. 7) calculated from Table I, and using (36) to fifth order and inserting either gain or phase contributions ( $a_i$  or  $b_i$ ) only. These two contributions add on a power basis from (39) to form the total predicted power dependence of  $M_{\alpha-\beta+\gamma}$  (heavy line). For power levels below +18 dBm,  $|M_{\alpha-\beta+\gamma}|$  is found to increase. This trend is also observed in the decrease of  $k(p_{\text{out}})$  at low levels (Fig. 6) but not taken into account by the present model. This behavior can be included, however, into an extended, coupled-mode model where the initial mode saturates at an appropriately low power level.

Noise-loading tests [20] were limited in the low-power region by low S/N ratios and at high power levels by low values of NPR [20], [23]. However, approximately the same dynamic range as in the earlier measurements was covered. The results were translated to  $M_{\alpha-\beta+\gamma}$  values using (45) and (47), and are also shown in Fig. 7 (solid points). Over most of the dynamic range, the NPR and  $M_{\alpha-\beta+\gamma}$  measurements agree to better than  $\pm 0.5$  dB. Although low NPR correction (Fig. 5) was applied, the agreement at levels above the 1-dB gain compression point is not as good. This is attributed to higher order deviations. At low levels the S/N ratio limitations are believed to be responsible for the observed deviations.

### VIII. CONCLUSIONS

When characterizing the IM performance of devices or systems, it is important to consider both amplitude and phase nonlinearities. A simple mathematical model representing both contributions is used to interrelate IM parameters obtained from four measurement techniques of varying complexity.

The single-tone test uses a bridge circuit to evaluate gain and phase nonlinearities separately. Because of the simple and well-defined nature of the signal, this test allows detailed characterization of the frequency and power level dependence of nonlinearities. The result is a set of gain and phase coefficients up to the desired order of approximation. When limited to third order and power levels well below saturation, the 1-dB gain compression point  $P_1$  [dBm], together with the AM-PM conversion constant  $k$  [ $^{\circ}/W$ ] represent good estimates of IM performance.

The two-tone test uses narrow BPF's to find the output power scattered into the selected IM product frequency by both gain and phase nonlinearities simultaneously. Measurements are made at low input power levels and extrapolated to

the intercept point  $P_I$  [dBm] between the fundamental and IM output power levels. For third-order nonlinearities,  $P_I$  is a convenient constant to compare IM performance of devices; when used in system design considerations, care has to be exercised to ensure that the extrapolations are still valid in the operating power range. This problem can be overcome by defining a two-tone third-order IM coefficient analogous to the one defined for three-tone tests.

The three-tone test requires improved filtering and/or signal tone cancellation to measure the power output at the selected IM product. The more even spectral distribution of the test signal over the desired band makes this test attractive for evaluating performance under multifrequency loading conditions. The third-order intermodulation coefficient  $M_3$  [dB] characterizes the sum of gain and phase nonlinearities. For third-order devices,  $M_3$  is a constant over a wide dynamic range; since it can be measured directly at the operating power level, it is a convenient measure for both device characterization and system design.

Noise loading uses a narrow BSF to remove a very small fraction (notch) from the band-limited white noise input spectrum. The power scattered into the notch is measured at the output to find the noise-power-ratio, NPR [dB], which characterizes the total IM performance. For third-order nonlinearities, the noise constant  $N_r$  [dB] is independent of power level over a wide dynamic range and, therefore, a convenient measure for both device and system performance under noiselike loading conditions.

Selection of a nonlinearity test method depends on the testing purpose: For detailed device characterization, the single-tone test yields the most specific diagnostic information; for device comparisons, the constants  $P_I$ ,  $M_3$ , and  $N_r$  are desirable; for system design, as was found in the CATV industry, test results become more reliable as the test signal spectrum approaches that of the system load. When devices are cascaded and if interactions between nonlinearities can be neglected, the influence of each section on overall performance can be evaluated. This is accomplished by referring the constants to the output of the cascade using the insertion gain (or loss) between the section and cascade outputs, raised to the appropriate power.

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## Broad-Band Microwave Measurements on GaAs «Traveling-Wave» Transistors

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**Abstract**—Instantaneous gain, noise figure, reverse attenuation, and gain and phase control measurements in the frequency range 8-18 GHz have been performed on GaAs traveling-wave transistors. The broad-band high-gain nature of the device together with the requirement for several bias connections precluded the use of standard test fixtures, and resulted in a package design exhibiting less than 1-dB insertion loss over the band together with 75- to 90-dB internal isolation. Untuned  $X$ -band gain, noise figure, and reverse attenuation were 12 dB, 18 dB, and 32 dB, respectively, and the gain and phase could be electronically varied over a 35-dB and 360° range. When RF tuning was employed, the gain, on the average, improved by 10 dB.

### I. INTRODUCTION

IN 1967 Robson *et al.* [1] published a concise description of a two-port amplifier that made special use of the growing space-charge waves which travel unidirectionally from cathode to anode in a slab of n-type GaAs biased above the transferred-electron threshold. The high internal gain and built-in isolation made the device potentially attractive, but the use of closely compensated bulk material forced the authors to use pulsed biasing and placed constraints on the geometry which limited the net gain to several decibels and made the gain fall off rapidly above a few gigahertz.

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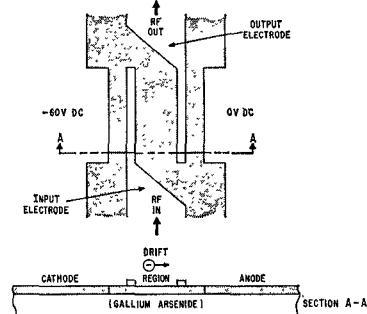


Fig. 1. Schematic of initial RCA traveling-wave transistor.

In 1970 Dean *et al.* [2] fabricated a similar device using 2- $\mu$ m-thick epitaxially grown n-on-insulating GaAs. Because of the use of purer epitaxial material, dc biasing could be employed, and a more favorable geometry was obtained (see Fig. 1). The geometry employed in the epitaxial device resulted in significant unidirectional net gain in  $X$  band (8-12 GHz). The RF coupling electrodes on this device acted as Schottky-barrier electrodes, and in subsequent work, described in a 1972 paper by Dean and Matarese [3], it was shown that the input portion of the device behaved very much like a field-effect transistor. An EM wave propagating on the input line, shown in Fig. 1, produces a voltage. This voltage drives a conduction current, which establishes a fluc-